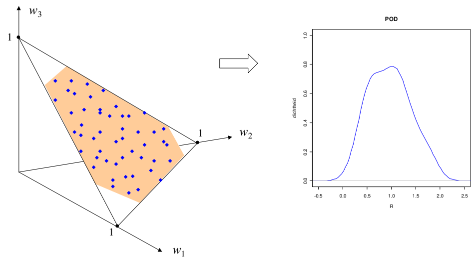
Portfolio Opportunity Distributions



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Master thesis (2006 - 2007)

Portfolio Opportunity Distributions (PODs):

- Technique for **Performance Evaluation**
- Rather unknown at the moment
- Makes use of computer simulations
- R. Surz et al. (1996)

What is Performance Evaluation?

Given:
- **Portfolio manager**
- **Mandate**

(set of portfolio construction rules; constraints)
- **Measurement period**
- **Performance data** (returns)

Main questions:
- **Is performance good?**
- Has the manager talent for this mandate?
- Hire / fire the manager?

(Quantitative data to support this decision)

Contents

- Traditional methods:
  - Benchmarks
  - Peer groups
- PODs: main idea
- Examples
- Technical remarks

Benchmark

Benchmark:
An index representing a financial market.

- Published market index
  (FTSE index, MSCI World, S&P 500, …)
- Customized index

Benchmark

Time series:
(monthly, quarterly, …)

$R_1^P, \ldots, R_n^P$: portfolio returns

$R_1^M, \ldots, R_n^M$: benchmark returns

$$R_i^E \quad := \quad R_i^P - R_i^M$$

$R_1^E, \ldots, R_n^E$: **excess returns**

Is performance good?

Performance is good
if
the manager has beaten the benchmark.

In other words: $\mathbb{E}R^E > 0$

Is performance good?

Statistical test: $H_0: \quad \mathbb{E}R^E \leq 0$
$H_1: \quad \mathbb{E}R^E > 0$

Reject the zero hypothesis at great values of

$$T_n := \sqrt{n}\,\frac{\overline{R_n^E}}{\sigma_{R^E,n}}$$

- R_i^E normally distributed: t-test.
- If not: approximate by CLT, or: use a sign test.

Problems with benchmarks

- Statistical significance must be obtained over time:

  1 observation per month $\Rightarrow$
  100 observations takes more than 8 years!

- Specific mandate rules are not reflected by the benchmark:

Problems with benchmarks

<u>Mandate A</u>:
European stocks
Benchmark: MSCI Europe
Tracking Error ≤ 10%
Max. exposure 5%

<u>Mandate B</u>:
European stocks
Benchmark: MSCI Europe
Tracking Error ≤ 15%
Max. exposure 5%

Both managers choose the same portfolio $\Rightarrow$
Performance score of both managers is equal.
This is quite unfair !

Problems with benchmarks

Mandate A:

Mandate B:

Range of possible returns:

Return: **2.6%** **2.6%**

Problems with benchmarks

- Start portfolio position gives side effects:

Mandate A:
European stocks
Benchmark: MSCI Europe
Tracking Error ≤ 15%
Max. exposure 5%

Start portfolio "badly" positioned

Mandate B:
European stocks
Benchmark: MSCI Europe
Tracking Error ≤ 15%
Max. exposure 5%

Start portfolio "luckily" positioned

Problems with benchmarks

Mandate A:

Mandate B:

Range of possible returns:

	Mandate A	Mandate B
Return PF:	**2.6%**	**2.6%**
Return BM:	**3.0%**	**3.0%**

Peer group

Peer group / manager universe:
Group of managers with a similar mandate over the same time period.

R^P : portfolio return

$R^{P_1},\ldots,R^{P_m}$: peer group returns

Idea: compare R^P with $R^{P_1},\ldots,R^{P_m}$.

Is performance good?

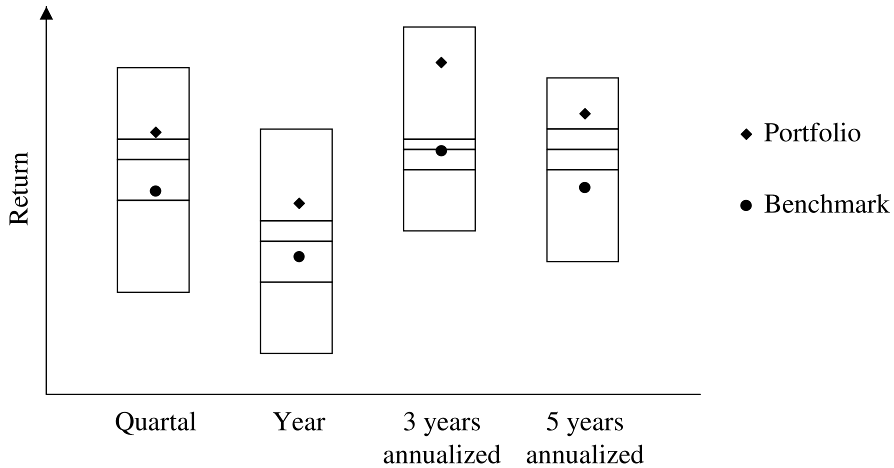
Performance is good
if
the manager has beaten his peers.

Is performance good?

$$\textbf{Quartile-ranking} := \begin{cases} 1 & R^P \text{ is in the top 25\%} \\ 2 & R^P \text{ is in the top 50\%} \\ 3 & R^P \text{ is in the top 75\%} \\ 4 & R^P \text{ otherwise} \end{cases}$$

Is performance good?

Floating bar chart:



Problems with peer groups

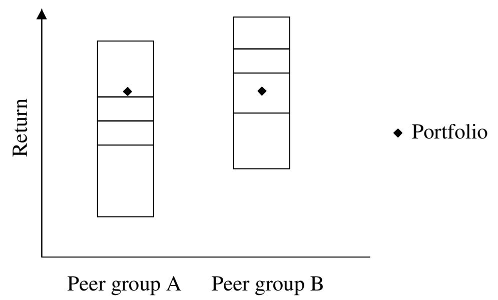
- **Survivorship bias**:
  Accounts terminated by underperformance are excluded from the database of the peer group provider.

  Longer measurement period $\Rightarrow$ data more biased.

  Marathon analogy (R. Surz, 1996):
  1000 runners,
  100 actually finish,
  is the 100th the last? Or in the top 10%?

Problems with peer groups

- **Classification bias**:
  When are mandates to be considered similar?
- **Composition bias**:
  Database provides too little observations:



Problems with peer groups

Eliminate **classification bias**:
Only consider mandates with (almost) the same constraints.
⇒ **composition bias**!

Eliminate **composition bias**:
Extend the universe by including mandates with (slightly) different constraints.
⇒ **classification bias**!

These “biasses” cannot both be eleminated.

Contents

Portfolio Opportunity Distributions

- An alternative for benchmarks and peer groups.
- Eliminates all flaws inherent in the classical methods.
- Basic idea: compare the realized return with all returns that *could have been* realized.

Portfolio Opportunity Distributions

Wall Street Journal, 1988:

Investors take the challenge to beat an imaginary gorilla (**market monkey**), which invests in a portfolio according to a random strategy.

Portfolio Opportunity Distributions

NOVA, Het Financiele Dagblad:

At the beginning of the year investment analists select a top-5 of the most promising dutch stocks for the upcoming year. Also, a delphin selects a top-5 of dutch stocks at random.

Portfolio Opportunity Distributions

Performance is good *if*
Manager beats the monkey / delphin!

Step 1

Specificy a **measurement period** and the manager's **mandate**:

Examples of mandate rules:

- Universe of investment securities.
- Bounds on exposure weights.
- Bounds on the number of different securities.
- Bounds on the weight in a certain class (region, country, sector,…)
- Bounds on ex-ante risk measures (tracking error).

Step 2

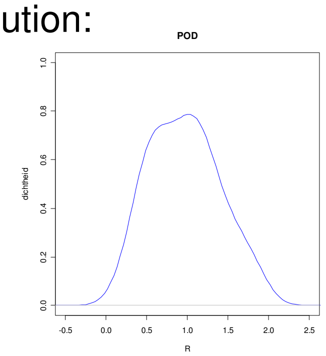
Simulate an investment manager by performing transactions at random.

The portfolio must satisfy the given mandate rules!

Generate a huge number (e.g. 1 000 000) of **random portfolios** which satisfy the manager's mandate.

Step 3

Compute the returns $R_1, \ldots, R_n$ of the sample portfolios.
This gives an estimation of a probability distribution:



Monte Carlo Simulation

Portfolio Opportunity Distrubution (POD)

Step 4

Apply Statistics:

R^P : realized return
$R_1,\ldots,R_n$: POD estimation
R : exact POD (stochastic variable)

POD ranking $\theta := \mathbb{P}(R > R^P)$

"probability that the monkey beats the investor"

POD ranking

Define: $Y_i := \begin{cases} 1, & R_i > R^P; \\ 0, & R_i \le R^P. \end{cases}$

Estimate: $\hat{\theta}_n := \overline{Y}_n = \frac{1}{n}\sum_{i=1}^{n} Y_i$

Confidence interval:

$$\theta = \overline{Y}_n \pm \sqrt{\frac{\overline{Y}_n(1-\overline{Y}_n)}{n-1}}\Phi^{-1}(1-\alpha/2)$$

Did you beat the monkey?

Statistical test:

$$H_0: \quad \text{median}(R) \geq R^P$$
$$H_1: \quad \text{median}(R) < R^P$$

- R_i normally distributed: t-test:

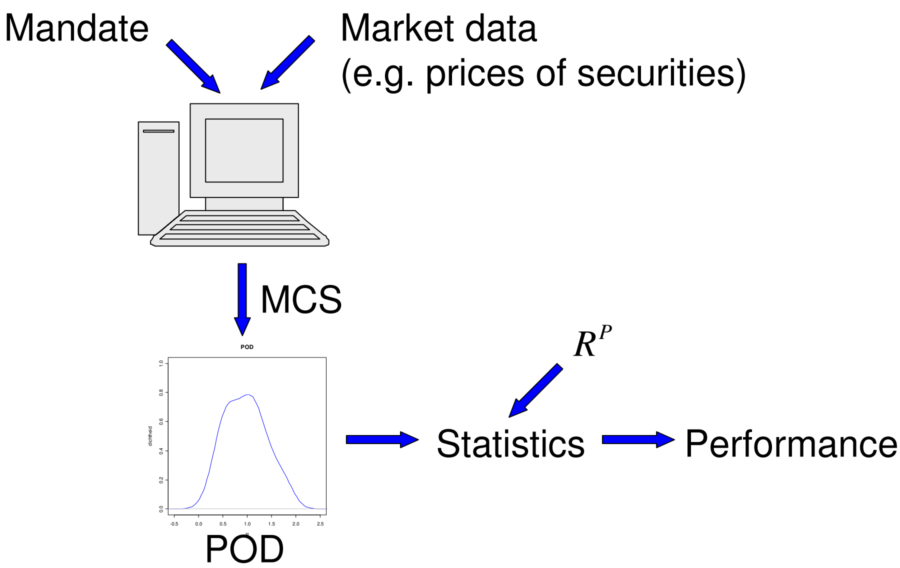
$$T_n := \sqrt{n}\,\frac{\overline{R}_n - R^P}{\sigma_{R,n}}$$

- If not: sign test:

$$T_n := \#\{i : R_i < R^P\}$$

Reject the zero hypothesis at great values of T_n .

Summary



Advantages w.r.t. benchmarks

- Per period a complete probability distribution instead of 1 numerical observation.
- All mandate rules are reflected, not only the investment universe.
- Start portfolio position is taken into account.

Advantages w.r.t. peer groups

- No classification bias.
- No composition bias.
- No survivorship bias.

POD universe can be considered a “perfect” peer group, **but:** consists of random portfolios, not realistic portfolios.

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Examples

Illustration of the POD technique with 2 example investment mandates:

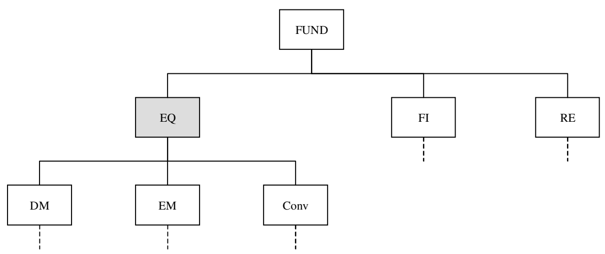
- Allocation to 3 equity segments.
- Selection out of 314 Japanese stocks.

Example 1

Equity segment of an asset management firm.

Allocation to 3 underlying segments:

- Developed Market (DM)
- Emerging Market (EM)
- Convertible bonds (Conv)



Example 1

Mandate rules:

- Ex-ante tracking error smaller or equal 5.5%.
- Exposure weights within these intervals
  - DM: [77.8%, 91.6%];
  - EM: [5.5%, 13.9%];
  - Conv: [2.8%, 8.4%].

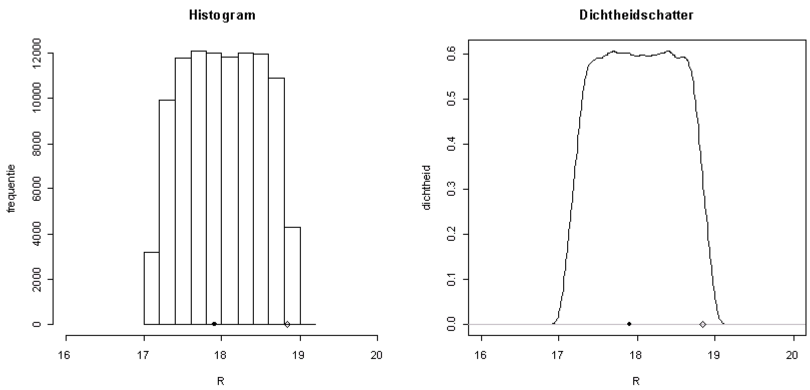
Measurement period: 2005-2006

Realized returns (ann.):

- PF: 18.84%
- BM: 17.90%

Example 1

Portfolio Opportunity Distribution



histogram density estimate

Example 1

- POD ranking:

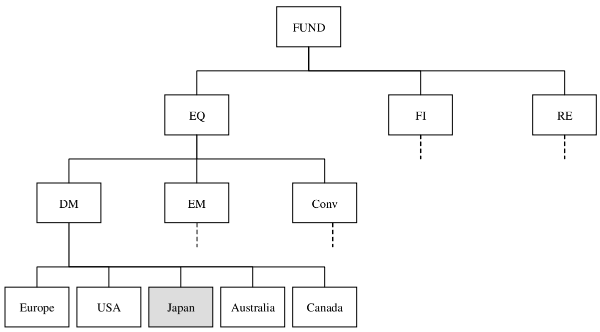
  $\hat{\theta}_n = 0.0291$, with 95% conf.intv. $\theta \in [0.0281, 0.0302]$

- Sign test:

  $p = 0$

Example 2

Selection mandate Japanese stocks.
Approximation: 314 MSCI Japan stocks.



Example 2

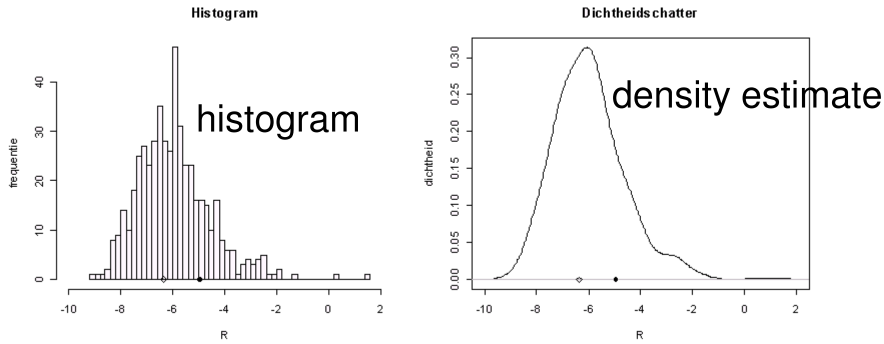
Mandate rules:
- Ex-ante tracking error smaller or equal 1.7%.
- Exposure weights within the interval [0%, 5%].
- Number of positive weights between 50 and 80.

Measurement period: 2006

Realized returns:
- PF: -6.359%
- BM: -4.959%

Example 2

Portfolio Opportunity Distribution



POD ranking: $\hat{\theta}_n = 0.584$, with 95% conf.intv. $\theta \in [0.541, 0.627]$

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How to simulate?

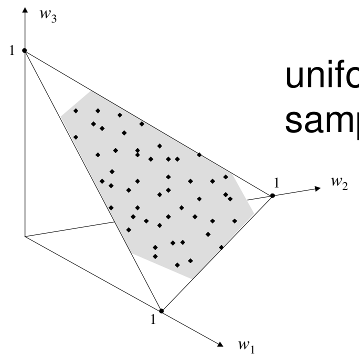
Two approaches:

- **Random start portfolio**
  (master thesis)
- **Random transactions**
  (new insight)

Random start portfolio

Select the start portfolio at random, and use buy-and-hold over the measurement period.

Unit simplex: feasible set corresponding to the manager's mandate



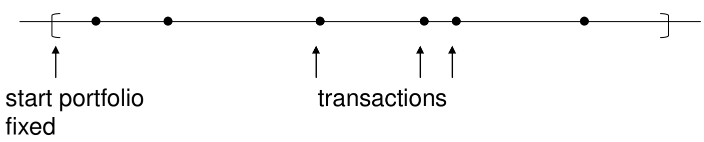
uniform sample

How to draw one random portfolio vector:
Markov Chain Monte Carlo

Random transactions

Take the start portfolio as it really is, and perform random transactions.
At each point in time the portfolio must satisfy the mandate's constraints.

Measurement period:



Critical notes

- Comparison with a "monkey", not with real competiting managers.
- Mandate must be well defined.
- Availability of input data may be problematic (transaction costs, liquidity, derivatives, …).
- Possibly a long run time is required by the computer simulation.

Extentions

- Determine the effect of a certain mandate rule.
  Compare the POD with the POD calculated after removing (or relaxing) the constraint.
- Fixed income portfolios:
  Do not use all securities, but just indices of duration/sector classes. This way, the return without selection effect can be evaluated.